

Computation of pion and kaon heavy ion multiplicities in a gluon-meson model

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In high energy Heavy Ion Collisions, the onset of the quark-gluon plasma is the colour glass condensate, dominated by gluons. The final state is hadronic, and dominated by pions and kaons. Here we investigate an effective approach of QCD with these bosonic fields and which can help to describe the transition of gluons into light mesons. Formally, our approach consists in integrating out the quark fields from the QCD path integral. In this way the fermionic fields are replaced by light mesons, such as the pions and sigma field. We apply our effective action to compute the number of pions and kaons per gluon emitted by a Boltzmann gluon gas, their multiplicities as a function of the gluon mass. We conclude that an effective gluon mass remains finite at $T = T_c$.

I. INTRODUCTION

In this work we develop a Lagrangian where only bosons, i. e. gluons and mesons, are the active degrees of freedom, and apply it to study the multiplicities in heavy ion collisions, the number of pions and kaons per gluon, and the gluon mass at the onset of the deconfinement/confinement phase transition [1].

Effective approaches of QCD have been widely used to study the properties of strong interactions [2]. Quark models, meson effective models, or models combining both quarks with mesons are used thoroughly to explore hadronic physics. Although gluons have been proposed already in the 70's together with the theory of strong interactions, QCD, in effective models it is common to assume that the gluons are integrated out, and only contribute indirectly through the quark or effective hadron interactions.

Nevertheless, there are two rapidly developing QCD domains where gluons are either easier to work with, or are phenomenologically more relevant. In many-body systems, the Grassmann variable nature of the quarks makes them technically much more difficult to address than the bosonic gluons. In particular, Lattice QCD first developed and applied pure gauge or quenched techniques since working with dynamical quarks is computationally very expensive [3, 4]. Moreover, in high energy Heavy Ion Collisions [1], it was proposed successfully that the onset of the quark-gluon plasma is the colour glass condensate, dominated by gluons [5–8]. The final state is hadronic, and dominated by pions and kaons. For instance, in the many particle BAMPS set-up for heavy ion collisions, [9, 10] the simulations are performed with gluons only form the onset, and mesons are included as final states of the hadronization. From the QCD perspective, in this

case, quarks and not gluons are integrated out and effective mesons are included. Note also the recent work of Weinberg [11] where an effective Lagrangian with gluons, in addition to pions and quarks, was put forward.

Formally, the different effective approaches to QCD can be seen as the result of integrating out some degrees of freedom from the QCD Lagrangian [2]. For instance, when only gluons are integrated out, one obtains a NJL-like theory [12] or a quark model [13]. If, in addition, also quarks are integrated out from the NJL model, one is left with a purely linear σ model. Moreover, as an intermediate step between the NJL and the σ models a quark-meson model is obtained. Similarly, in the approach of Cahill and Roberts [14, 15] it was shown by using bilocal auxiliary fields (along the same line of the Hubbard-Stratonovich transformation [16]) how to integrate out quark and gluon degrees of freedom to obtain a purely mesonic Lagrangian. While these calculations were only performed at one loop order, it was an interesting approach to connect effective models, say the σ model, directly to QCD. In addition, there are also lattice QCD approaches for effective meson theories [17], which deliver qualitatively similar results.

For the purpose of this paper it is necessary to chose a slightly different way, which consists of integrating out from the QCD Lagrangian the quarks only and then obtain a gluon-meson (fully bosonic) theory. From symmetry principles (colour gauge invariance and chiral symmetry) we expect at leading order the following tree-level coupling between the gluons, the pions and their chiral partner, the scalar σ meson:

$$\mathcal{L}_{\text{gluon-meson}} \propto (G^{a,\mu\nu} G_{\mu\nu}^a)(\sigma^2 + \vec{\pi}^2 + \dots), \quad (1)$$

where $G_{\mu\nu}^a$ is the gluonic field tensor and dots refer to other mesonic degrees, such as resonances with open and hidden strange-quark content (such as the kaons), vector resonances and, eventually, non-quarkonium resonances.

The interaction Lagrangian (1) allows to study the transition of two gluons into a couple of mesons. If the latter are not stable, they further decay into pions or

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kaons. For the mesonic sector we apply the most comprehensive σ model, including not only scalar and pseudoscalar mesons, but also vector and axial-vector mesons [18, 19]. By assuming a thermal bath for gluons at the hadronization point, we can then evaluate how many pions (and kaons) are obtained per gluon as function of the temperature T of the bath. To account for a possible finite scale with an energy dimension, in the gluon sector at finite T , we allow for a finite effective gluon mass in the Boltzmann distribution.

The paper is organized as follows: in Sec. II we derive our effective Lagrangian, in Sec. III we present the analytical and numerical results, and in Sec. IV we derive our conclusions.

II. THE GLUON-MESON INTERACTION

In this section we present the formal steps necessary to obtain a gluon-meson theory. We start from the euclidean QCD generating functional

$$Z[\bar{\eta}, \eta] = \int \mathcal{D}\bar{q}\mathcal{D}q\mathcal{D}A\mathcal{D}\bar{\omega}\mathcal{D}\omega \times e^{-\int_x \mathcal{L}_{QCD} + \mathcal{L}_{GF} + \mathcal{L}_{FPG} - \bar{\eta}q - \bar{q}\eta}, \quad (2)$$

where

$$\mathcal{L}_{QCD} = \bar{q}(\gamma_\mu D^\mu + m)q + \frac{1}{4}G_{\mu\nu}^a G_{\mu\nu}^a, \quad (3)$$

$$D_\mu = \partial_\mu - iA_\mu, \quad A_\mu = gA_\mu^a \frac{\lambda^a}{2}.$$

$\bar{\eta}$ and η are the fermion sources, \mathcal{L}_{GF} and \mathcal{L}_{FPG} represent the gauge-fixing and the Faddeev-Popov ghost terms. In a suitable gauge [14, 15], and with no fermion sources, the generating functional of QCD can be written as

$$Z = \int \mathcal{D}\bar{q}\mathcal{D}q\mathcal{D}A e^{-\int_x \bar{q}(\gamma_\mu D^\mu + m)q - \frac{1}{2}\int_x A_\mu^a (D^{-1})_{\mu\nu}^{ab} A_\nu^b}, \quad (4)$$

where

$$D_{\mu\nu}^{ab}(x-y) = \int \mathcal{D}\bar{\omega}\mathcal{D}\omega\mathcal{D}A A_\mu^a(x) A_\nu^b(y) \times e^{-\int_x \frac{1}{4}G_{\mu\nu}^a G_{\mu\nu}^a + \mathcal{L}_{GF} + \mathcal{L}_{FPG}}$$

is the exact gluon propagator which contains all gluon self-interactions and gluon-ghost interactions but excludes quark loops.

To introduce the mesons we multiply Eq. (4) by

$$1 = \frac{1}{\mathcal{N}} \int \mathcal{D}\Phi\mathcal{D}R_\mu\mathcal{D}L_\mu e^{-\int_x \mathcal{L}_{\text{meson}}}, \quad (6)$$

where

$$\mathcal{L}_{\text{meson}} = \text{Tr} [\partial^\mu \Phi^\dagger \partial_\mu \Phi] - m_0^2 \text{Tr} [\Phi^\dagger \Phi] - \frac{1}{4} \text{Tr} [R_{\mu\nu}^2 + L_{\mu\nu}^2] + \frac{m_1^2}{2} \text{Tr} [R_\mu^2 + L_\mu^2] \quad (7)$$

is the quadratic part of the globally invariant linear σ model Lagrangian with $U(N_f)_R \times U(N_f)_L$ symmetry: Φ , L_μ and R_μ are $N_f \times N_f$ Hermitian matrices for the (pseudo)scalar, vectorial right-handed and left-handed mesonic degrees of freedom [19]:

$$\begin{aligned} \Phi &= S^a t^a + iP^a t^a, \\ L_\mu &= L_\mu^a t^a = V_\mu + A_\mu, \\ R_\mu &= R_\mu^a t^a = V_\mu - A_\mu, \\ R^{\mu\nu} &= \partial^\mu R^\nu - \partial^\nu R^\mu, L^{\mu\nu} = \partial^\mu L^\nu - \partial^\nu L^\mu. \end{aligned} \quad (8)$$

In the previous expressions t^a are the N_f^2 generators of $U(N_f)$, $S^a = \sqrt{2}\bar{q}t^a q$ are the scalar, $P^a = \sqrt{2}\bar{q}i\gamma^5 t^a q$ are the pseudoscalar degrees of freedom, $V_\mu^a = \sqrt{2}\bar{q}\gamma_\mu t^a q$, $A_\mu^a = \sqrt{2}\bar{q}\gamma_\mu \gamma^5 t^a q$ are the vector and axial-vector microscopic quark currents. For instance, in the case $N_f = 2$ the fields are given by

$$\Phi = (\sigma + i\eta_N) t^0 + (\vec{a}_0 + i\vec{\pi}) \vec{t}, \quad (9)$$

where $t^0 = I/2$, $t^i = \tau_i/2$, and τ_i are the Pauli matrices. Analogously, $R^\mu = (\omega^\mu - f_1^\mu) t^0 + (\vec{\rho}^\mu - \vec{a}_1^\mu) \vec{t}$ represents the vector and $L^\mu = (\omega^\mu + f_1^\mu) t^0 + (\vec{\rho}^\mu + \vec{a}_1^\mu) \vec{t}$ the axial vector degrees of freedom. The extension to the case $N_f = 3$ is straightforward [20].

Clearly, upon Gaussian integration, Eq. (6) gives a constant factor and thus does not change the path integral in the generating functional Eq. (4).

To couple the mesons to fermions, the mesonic fields can be added as parallel transports in the fermion matrix similar to a mass term in the case of the scalar sigma and to a chiral transport in the case of the pion. The corresponding minimal coupling is given by

$$\gamma_\mu D^\mu \rightarrow \tilde{D} = \gamma_\mu D^\mu - c_1 \Phi - c_2 (\gamma_\mu V^\mu + \gamma_\mu \gamma^5 A^\mu), \quad (10)$$

where c_1 and c_2 are free parameters, and A^μ is the axial current. Finally, performing the Grassmann integration over the fermion fields, we obtain a purely bosonic theory

$$Z = \int \mathcal{D}\Phi\mathcal{D}L_\mu\mathcal{D}R_\mu\mathcal{D}A e^{-\int_x \frac{1}{2}A_\mu^a (D^{-1})_{\mu\nu}^{ab} A_\nu^b + \mathcal{L}_{\text{meson}}} \det[\tilde{D}].$$

The coupling of the mesonic and gluonic degrees of freedom resides in the fermion determinant $\det[\tilde{D}]$, which cannot be computed analytically. However, by requiring local colour gauge invariance and chiral symmetry and restricting to lowest dimensionality of the interaction Lagrangian, we are left to the following Lagrangian,

$$\mathcal{L}_{\text{gluon-meson}} = G^{a,\mu\nu} G_{\mu\nu}^a \text{Tr} [a\Phi^\dagger \Phi + b(V_\delta^2 + A_\delta^2)], \quad (11)$$

where a and b are couplings with dimension Energy^{-2} and describe the transition from two gluons to two mesons. To obtain the relation between the parameters a , b and the parameters c_1 , c_2 introduced in Eq. (10)

meson	M_m	g	N_π	N_K	f
π	138	3	1	0	a
η	549	1	3	0	a
η'	958	1	3	0	a
K	495	4	0	1	a
ρ	775	9	2	0	b
ω	782	3	3	0	b
ϕ	1020	3	0	2	b
K^*	892	12	1	1	b
a_0	985	3	4	0	c
σ	600	1	2	0	c
f_0	980	1	2	0	c
κ	800	4	1	1	c
a_0	1450	3	4	0	a
f_0	1370	1	4	0	a
f_0	1710	1	0	2	a
κ	1430	4	1	1	a
a_1	1230	9	3	0	b
f_1	1282	3	4	0	b
f_1	1420	3	1	2	b
K_1	1272	12	1	1	b

TABLE I. The parameters for each meson pair initially produced and then decaying into pions and kaons are the mass, degeneracy, number of pions produced by the meson, number of kaons produced by the meson, the family factor. For meson octets and their respective chiral partners we assume the same family factor.

it would be necessary to evaluate the fermion determinant $\det[\tilde{D}]$ analytically. Although it is not possible to compute the fermion determinant exactly, it is natural to expect that $a \sim b$. In the following we will work with the simplified assumption $a = b$. Note also that the Lagrangian (11) makes use of the so-called flavour blindness of the gluon fields.

In accordance with Parganlija et al. [19], the scalar partners of the pions are identified with the scalar resonances above 1 GeV. However, for completeness, one should also include the scalars below 1 GeV, which according to many recent and less recent studies are non-quarkonium states. In fact, these states can be enhancements in the two-pseudoscalar channel or tetraquark states, see Refs. [2] and references therein for a more detailed discussion of this point. Here they are coupled to the gluons with an independent coupling c . We shall here test two choices: $c = a$, i.e. with equal strength as the other channels, and $c = 0$, where the light scalars are switched off.

The explicit evaluation of the traces delivers

$$\begin{aligned}
\mathcal{L}_{\text{gluon-meson}} = & a G^{a,\mu\nu} G_{\mu\nu}^a (\vec{\pi}^2 + \dots + \vec{a}_0^2 + \dots) \\
& + b G^{a,\mu\nu} G_{\mu\nu}^a (\vec{\rho}^\delta \cdot \vec{\rho}^\delta + \dots + \vec{a}_{1\delta} \cdot \vec{a}_{1\delta} + \dots) \\
& + c G^{a,\mu\nu} G_{\mu\nu}^a (\sigma^2 + \dots), \quad (12)
\end{aligned}$$

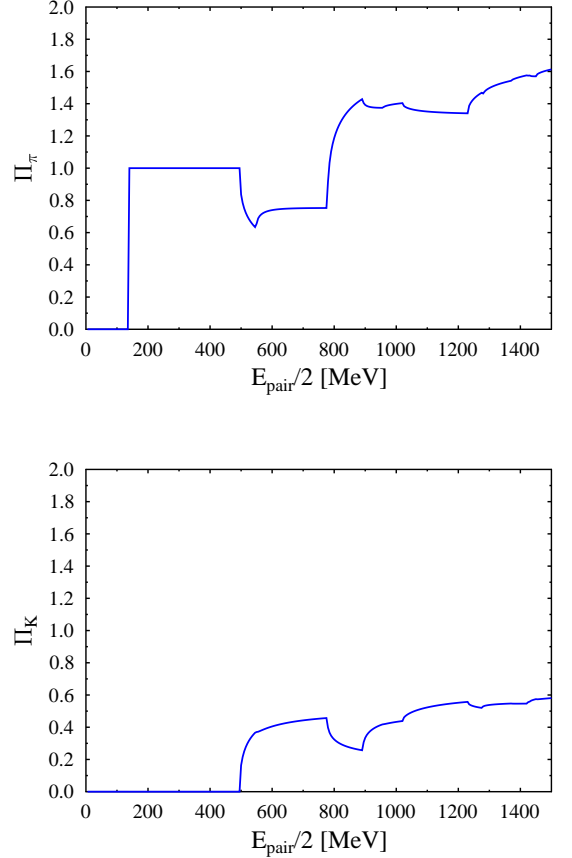


FIG. 1. Number of (top) pions $\Pi_\pi(E_{pair})$ and of (bottom) kaons $\Pi_k(E_{pair})$ produced per gluon as a function of the gluon energy in the centre of mass of the gluon pair. Here $a = b$, $c = 0$ are used.

where dots refer to further quadratic mesonic interactions listed in Table I.

Even if higher dimensionality terms in the gluon-meson Lagrangian exist, with more derivative or field terms, the reaction from two gluons to two mesons should dominate during the freeze-out at the boundary of the gluon plasma. Thus we employ this Lagrangian (12) to evaluate the number of produced pions and kaons per gluon.

III. ANALYTICAL AND NUMERICAL RESULTS

In our framework an effective gluon mass M_g can be introduced. Notice the existence of a possible effective gluon mass [21], or pole in a gluon propagator, already at $T = 0$, has been under debate in QCD. While gauge invariance in a perturbative approach rules out a gluon mass and the existing lattice QCD glueballs suggest that the gluon propagator is transverse, nevertheless a run-

ning gluon mass does not contradict gauge invariance [22], and there is also evidence both from Landau Gauge lattice QCD [23] and from the glueball spectrum for a finite pole in the gluon propagator. Most results, several of them from lattice QCD calculations, point to a $T = 0$ effective gluon mass or to other possible scales, say an effective dual gluon mass [24], in the range [0.5, 1.0] GeV. A possible gluon mass at finite T is also starting to be investigated in lattice QCD [25]. Moreover in the BAMPS set-up a finite Debye mass [9] for the gluon is also considered,

$$M_g^2 = \frac{24}{\pi} \alpha_s T^2, \quad (13)$$

which, say at $T = T_c = 0.158$ GeV and $\alpha_s \simeq 0.3$ leads to a gluon mass of $M_g \simeq 0.239$ GeV.

For the following purposes we evaluate the Lorentz-invariant Mandelstam variable s for a system of two gluons with four-momenta

$$p_i = (\sqrt{\mathbf{p}_i^2 + M_g^2}, \mathbf{p}_i), \quad i = 1, 2. \quad (14)$$

The Mandelstam variable s leads to the centre of mass energy E_{pair} of the gluon pair,

$$\begin{aligned} s &= (p_1 + p_2)^2 \\ &= 2M_g^2 + 2 \left(\sqrt{\mathbf{p}_1^2 + M_g^2} \sqrt{\mathbf{p}_2^2 + M_g^2} - \mathbf{p}_1 \cdot \mathbf{p}_2 \right) \\ &= E_{pair}^2. \end{aligned} \quad (15)$$

Due to the Lagrangian in Eq. (12), the two gluons convert into a meson-pair. Considering that only pions and kaons are regarded as stable, we must also take into account that the other resonances decay subsequently into kaons and pions. For instance, each σ meson decays into a two-pion pair, therefore the $\sigma\sigma$ channel results into a final 4π state. Similarly, a pair of ρ mesons decays also into four pions. On the contrary, the vector state ϕ decays predominantly into kaons. In Table I these conversion factors, expressed as $n^{(\pi)}$ and $n^{(K)}$, are listed for all the relevant mesons used for the calculations. Moreover, we also include the usual phase space factor $\sqrt{\frac{E_{pair}^2}{4} - M_k^2}$, where M_k is the mass of the k -th meson pair, and the corresponding degeneracy spin-isospin factor g_k . Finally, we should also take into account the relative strength, which is equal to a^2 in the (pseudo)scalar channel, to b^2 in the (axial-)vector channel, and to c^2 in what concerns the scalar channel below 1 GeV. Putting all together, the number of pions and kaons per gluon is calculated as

$$\Pi_i(E_{pair}) = \frac{\sum_k \sqrt{\frac{E_{pair}^2}{4} - M_k^2} \theta(E_{pair} - 4M_k) g_k f_k^2 n_k^{(i)}}{\sum_k \sqrt{\frac{E_{pair}^2}{4} - M_k^2} \theta(E_{pair} - 4M_k) g_k f_k^2} \quad (16)$$

where $i = \pi, K$.

The functions $\Pi_\pi(E_{pair})$ and $\Pi_K(E_{pair})$ as a function of the gluon energy and as a function of the Boltzmann temperature are both depicted in Fig. 1.

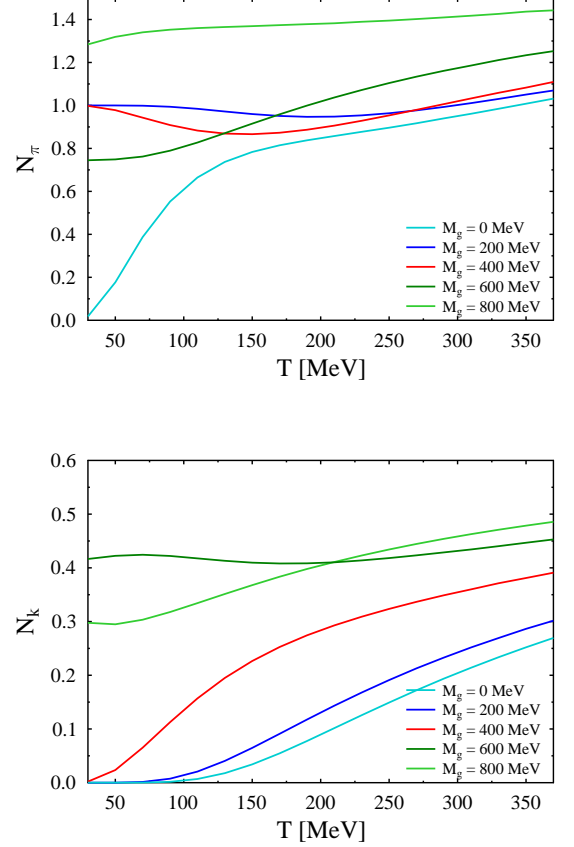


FIG. 2. Number of (top) pions and (bottom) kaons produced per gluon as function of the temperature T for different gluon masses.

We are interested in comparing our results with the parametrization of the BAMPS set-up [9], since, like our framework, BAMPS also consists of gluons, decaying into pions. In BAMPS, 1.5 to 2.0 pions are produced per gluon pair. This production takes place in different conditions than ours, at non-chemical equilibrium with local and dynamical many-gluon simulations. Nevertheless, if we compare with our approach, we notice we can easily obtain $N_\pi \sim 1$, but the increase of N_π above unity can only be achieved at the price of including a sizeable effective gluon mass. This confirms the importance of including in our framework an effective gluon mass, simulating a finite non-perturbative scale characteristic of the gluon plasma.

We now evaluate the emitted number of pions $N_\pi(T)$ and kaons $N_K(T)$ per gluon as a function of the temperature T . We denote the multiplicities $N_i(T)$ by employing a Boltzmann distribution of each gluon, thus leading to

$$N_i(T) = \int d\mathbf{p}_1 d\mathbf{p}_2 f_B(\mathbf{p}_1^2, T) f_B(\mathbf{p}_2^2, T) \Pi_i(E_{pair}), \quad (17)$$

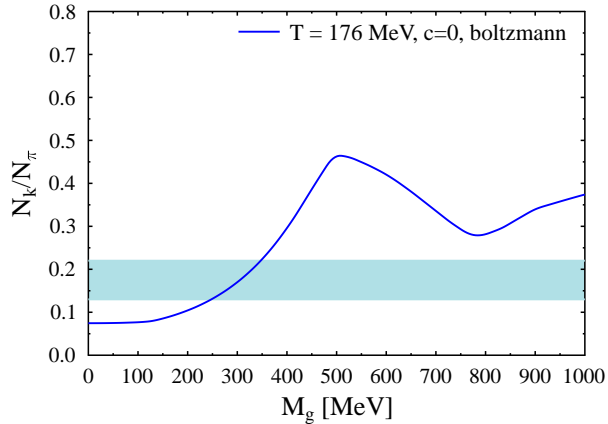


FIG. 3. We show (solid line) our result and (band) the BNL, CERN and FNAL data for the kaon to pion multiplicity ratio as a function of a possible gluon mass, at $T = T_c \simeq 158$ MeV. The measured ratio $N_K/N_\pi \in [0.13, 0.22]$ is realized for a gluon mass $M_g \in [0.25, 0.35]$ GeV.

where $f_B(\mathbf{p}^2, T) = \mathcal{N} e^{-\sqrt{\mathbf{p}^2 + m_{gluon}^2}/T}$ is the normalized Boltzmann distribution. The integral in Eq. () can be simplified to a three-dimensional integration, and we compute it with a numerically accurate c++ code. In Fig. 2 the functions $N_\pi(T)$ and $N_K(T)$ are plotted for different values of the gluon mass M_g and for both choices $a = b$, $c = 0$ and $a = b = c$. For the intermediate value $M_g = 400$ MeV we have roughly one pion per each gluon for each T , while $N_K(T)$ is a rapidly increasing function with T .

In Fig. 3 we present the ratio $N_K(T)/N_\pi(T)$ for the temperature of $T = T_c = 0.158$ GeV as a function of the gluon mass. The critical temperature T_c for the confinement and chiral crossover was measured in lattice QCD to be in the range $T_c \in [0.145, 0.165]$ GeV [26–29]. This was achieved in very precise full lattice QCD simulations with dynamical fermions. This temperature range is consistent with the freeze-out temperature of the quark-gluon plasma measured in Heavy ion collisions. The freeze-out temperature in heavy ion collisions can be determined from the inverse slope of the hadronic species multiplicity as a function of the transverse momentum. Recent analysis of heavy ion collisions indicate that the freeze-out temperature is in the range of $[0.150, 0.170]$ GeV with results between 0.150 GeV and 0.160 GeV [30] and results between 0.160 GeV and 0.170 GeV [31, 32]. Both ranges are compatible, and we consider in our computations the mean value of $T = 0.158$ GeV.

In Fig. 3 we compare the ratio with the experimental data $(N_K/N_\pi)_{\text{exp}} \in [1.13, 1.22]$ measured by the PHENIX, STAR, BRAHMS, E866 and NA49 collaborations and extrapolated by the UrQMD 2.0, UrQMD 2.1 and HSD transport approaches [32, 33] for the ratio of the

pion and kaon multiplicity in the most central collisions.

Our results point to a solution corresponding to a possible effective gluon mass, in a range of $M_g \in [0.28, 0.37]$ GeV. Remotely, a second less likely mass of circa 0.8 GeV may be possible. We notice that the solution points to a gluon mass at T_c of the order of circa 0.4 of the gluon effective mass of 0.5 to 1.0 GeV at $T = 0$ resulting from different gluon calculations. The solution for the gluon mass is also consistent with the Debye mass [9] of the gluon at finite T .

Finally we test the robustness of our results checking the parameter dependence of the gluon-meson model. In Fig. 4 we compare the ratio N_K/N_π for different couplings ($c = 0$ with $c = 1$), different temperatures ($T = 0.145$ GeV with $T = 0.170$ GeV) and different statistical distributions (Boltzmann and Bose-Einstein). All the tests we performed with plausible changes of our parameters suggest our results are robust.

The small effect of changing the coupling of gluons to mesons is quite relevant for our results, since the meson properties are not yet established. Strong coupled channel effects, tetraquarks, and glueballs have been shown to affect the meson spectrum and the mesonic couplings. Here we utilize the meson properties listed in the Particle Data Group [34], but other meson data would yield similar results. We utilize the sigma model for the meson production, but other hadronic models would again yield similar results.

The robustness of our results occurs because a gluon in the freeze-out of the plasma has a low energy ($T \simeq 160$ MeV and an effective mass of $M_g \simeq 320$ MeV), much lower than the energy of a gluon in any glueball typical of lattice QCD simulations or of constituent gluon model estimations. Thus our results do not really depend on the details of the meson spectrum and of the meson couplings above that low energy, and escape the problem of understanding higher energy reactions such as the glueball decays.

IV. CONCLUSIONS

In this work we develop an effective Lagrangian which connects gluons to mesons. We then utilize this approach to calculate the emitted number of pions and kaons per gluon out of a gluon gas at temperature T . We assume the gluons are in a thermal bath at $T = T_c$ whereas the mesons are produced at vanishing temperature.

The fact that our effective Lagrangian consists of bosonic degrees of freedom only might be also interesting for lattice QCD applications. In fact, the Grassmann variable nature of the quarks makes them technically much more difficult to address (see for instance Ref. [3] and refs. therein) than the bosonic gluons and mesons.

Our approach represents an attempt to link directly and in an understandable way the gluon-dominated physics of the quark gluon plasma, as suggested by the colour glass condensate and BAMPS approaches, to the

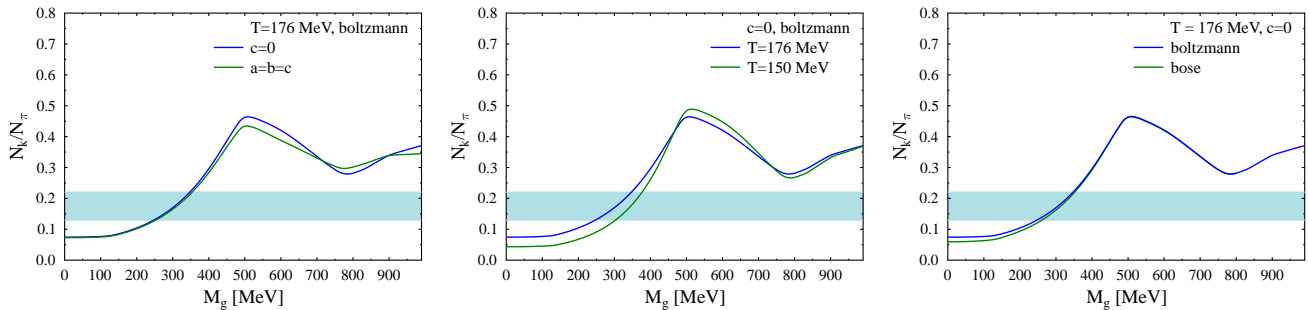


FIG. 4. The parameter dependence in the gluon-meson model. We compare the ratio N_K/N_π for different couplings (left, $c = 0$ with $c = 1$), different temperatures (centre, $T = 0.145$ GeV with $T = 0.170$ GeV) and different statistical distributions (right, Boltzmann and Bose-Einstein).

light hadrons in the final stage.

Further developments of our approach could include meson mass modifications in the medium, effects of the freeze-out boundary, different meson-gluon couplings, missing resonances such as the tensor mesons, final state interactions among mesons, and, last but not least, glue-ball fields, which directly couple to gluonic fields. Nevertheless, all the tests we performed with plausible changes of our parameters suggest that our results are stable. Thus we regard our effective Lagrangian as a pilot study toward a better understanding of the rich physics of QCD and of Heavy Ion Collisions.

Interestingly, to reproduce the experimental results of the PHENIX, STAR, BRAHMS, E866 and NA49 collaborations for the pion and kaon multiplicities, we have to include a finite scale for the gluon energy at $T = T_c$. Our results point to a possible effective gluon mass, in a range of M_g between 0.28 and 0.37 GeV if we consider the uncertainty on the rapidity ratio, or of $M_g \in [0.25, 0.39]$ if we also consider the uncertainty on the freeze-out temperature. We notice that this solution points to a gluon mass at T_c of the order of circa 0.4 and 1.0 of the gluon effective mass of 0.5 to 1.0 GeV at $T = 0$ resulting from different gluon calculations. An $M_g \in [0.25, 0.39]$ is also

close to the Debye gluon mass at $T = T_c$. Notice this solution corresponds to an absolute pion multiplicity of circa 0.9 pions per gluon.

If the gluon mass is related to confinement, say as in superconductors, our result is consistent with the QCD order parameters at T_c . Both for a first order phase transition where the order parameter is discontinuous (found in quenched lattice QCD [35]) and in a crossover where the order parameter remains finite (found in lattice QCD with dynamical fermions [26]), the scale of confinement should not simply vanish at $T = T_c$. Here we present an evidence, based in the experimental Heavy Ion data and in our simple and robust gluon-meson model that a finite scale of 0.25 to 0.39 GeV exists in the gluon sector of QCD at $T = T_c$.

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